# Kinematic and Dynamic Analysis of the Moving Jaw for Single-toggle Jaw Crusher Design 

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#### Abstract

The research paper titled "Kinematic and dynamic analysis of the moving jaw for single-toggle jaw crusher design" primarily focuses on the kinematic and the force analysis to find out various dynamic forces acting at various elements of the moving jaw and the toggle plate. The paper also draws out the mathematical formulation on the analytical approach to predict the velocity and the acceleration of any point rigidly fixed to the moving jaw much needed for the inertial force calculation at various section as a function of different angle of the crank.. The mathematical model successfully can be used for the forces exerted on the moving jaw by the toggle plate and the force exerted by the crank.


## 1. INTRODUCTION

A jaw crusher consists of a set of vertical jaws, one jaw being fixed and the other being moved back and forth relative to it by a cam or pitman mechanism. The jaws are farther apart at the top than at the bottom, forming a tapered chute so that the material is crushed progressively smaller and smaller as it travels downward until it is small enough to escape from the bottom opening. The movement of the jaw can be quite small, since complete crushing is not performed in one stroke. The inertia required to crush the material is provided by a weighted flywheel that moves a shaft creating an eccentric motion that causes the closing of the gap. Single and double toggle jaw crushers are constructed of heavy duty fabricated plate frames with reinforcing ribs throughout.


Sectional view showing Components of a Jaw Crusher

## 2. KINEMATIC ANALYSIS OF JAW CRUSHER

Jaw crusher can be considered as a four bar mechanism in which, link OA (Fig. 2.1) is the crank, AB is the moving jaw and BC is the toggle bar. OC is the fixed link. To completely specify a four bar mechanism at any given instant, the length of all the four links and an angle should also be known. The known angle in this case is the angle of crank rotation $\beta$ measured in anticlockwise sense from H axis (shown in fig).

The $\mathrm{X}-\mathrm{Y}$ coordinate axis has its origin at O .The $\mathrm{U}-\mathrm{V}$ coordinate axis has its origin at B and is inclined at an angle $\alpha$ clockwise to the $\mathrm{X}-\mathrm{Y}$ axis. $\mathrm{X}^{\prime}-\mathrm{Y}^{\prime}$ is a coordinate axis having origin at A' and parallel to the $\mathrm{X}-\mathrm{Y}$ axis.

Center of the shaft is O . The eccentricity of the point A is $\mathbf{e}$. The extreme end of the toggle plate attached to the body with is C (a,-b). The length of the pitman is $\mathbf{l}$. Length of the toggle plate is $\mathbf{t}$.


Fig. 2.1

Using simple coordinates geometry and for typical values of l,t and e ( $1200 \mathrm{~mm}, 350 \mathrm{~mm}, 10 \mathrm{~mm}$ respectively), the values of $\alpha$ for corresponding values of $\beta$ are as in table 1.1.
$\left.\begin{array}{|l|l|l|l|l|l|l|l|l|l|l|l|l|}\hline \mathrm{B} & 0 & 30 & 60 & 90 & 120 & 150 & 180 & 210 & 240 & 270 & 300 & 330 \\ \hline \mathrm{~A} & \begin{array}{l}12.8 \\ 0\end{array} & \begin{array}{l}12.8 \\ 8\end{array} & \begin{array}{l}12.7 \\ 8\end{array} & 12.5 & 12.2 & 11.9 & 11.6 & 11.5 & 11.6 & 11.9 & 12.2 \\ 2\end{array}\right)$

Table 2.1 variation of angle between the moving jaw And Yaxis ( $\alpha$ ) and crank angle ( $\beta$ )

The coordinates of a point P rigidly attached to pitman is (u,v).Upon transforming it to $\mathrm{X}, \mathrm{Y}$ axis (having derived $\alpha$
As a function of $\beta$ ).

$$
\begin{align*}
& x=u \cos (\alpha)-(l-v) \sin (\alpha)+e \cos (\beta)  \tag{1.1}\\
& y=-u \sin (\alpha)+(v-l) \cos (\alpha)+e \sin (\beta) \tag{1.2}
\end{align*}
$$

The coordinate of the midpoint of pitman ( $\mathrm{x}, \mathrm{y}$ ) has been
Plotted as in Fig. 1.3.Similar procedure(Using eqn 1.1 and 1.2 ) was repeated after having divided the pitman body into six equal parts ( $\mathrm{p} 1, \mathrm{p} 2, \mathrm{p} 3, \mathrm{p} 4, \mathrm{p} 5$ and p 6 ) from the bottom point towards the top point. The locus of these points was found. The results have been plotted for points p1 and p6. (Fig2.3 and Fig. 2.4 respectively).From the locus of these points their vertical as well as horizontal throw was plotted (Fig. 2.5 and 2.6 respectively)


Fig. 2.2


Fig. 2.3


Fig. 2.4


Fig. 2.5


Fig. 2.6

## Conclusions

As we move upwards along the moving jaw i.e. along p1 to p6: -

1. The shape of the locus changes from the ellipse towards circle.
2. The vertical Throw decreases
3. The horizontal swing first decreases and then increases (in our case minimum occur at p3).

## 3. ANALYTIC EQUATION OF VELOCITY,

 ACCELERATION DIAGRAM.

Fig. 3.1.
The Fig. above shows the velocity diagram at any crank angle $\beta$..oa represents the crank velocity, ab the relative velocity of pitman wrt to the crank, and ob the toggle plate velocity. $\beta, \alpha, \mathrm{t}, \mathrm{l}$ and $\gamma$ are as per Fig. 2.1.
The coordinates of point a are $(\omega \mathrm{r} \cos (90+\beta)$, $\omega \mathrm{r} \sin (90+\beta))$.
Noting that slope of the line ob is $(90+\gamma)$ and the Line $a b$ makes angle $\alpha$ (clockwise) with respect to X-axis. Using coordinate geometry for the point of intersection of two lines and taking and Taking $(\mathbf{9 0}+\boldsymbol{\beta})=\boldsymbol{\varphi}$, (For simplification)
$\mathbf{V}_{\mathbf{b x}}=\{\omega \mathrm{r}(\cos \varphi \tan \alpha+\sin \varphi)\} /(\tan \theta+\tan \alpha) \mathbf{V}_{\mathbf{b y}}=\{\omega \mathrm{r}$ $\tan \theta(\cos \varphi \tan \alpha+\sin \varphi)\} /(\tan \theta+\tan \alpha)$

Where, $\mathbf{V}_{\text {bx }}$ and $\mathbf{V}_{\text {by }}$ are x and y components of velocity of point b.

The angular velocity of the pitman, $\omega_{\mathrm{p}}$, can be written as:-
$\omega_{\mathrm{p}}=\left[\left(\mathrm{V}_{\mathrm{bx}}-\omega \mathrm{r}(\cos \varphi)\right)^{2}+\left(\mathrm{V}_{\mathrm{by}}-(\omega \mathrm{r} \sin \varphi)\right)^{2}\right]^{(1 / 2)} /$


Fig. 3.2

The Fig. above shows the acceleration diagram at crank angle $\beta$. o'a' represent crank acceleration, a'b' the relative acceleration of pitman wrt crank and o'b' toggle plate acceleration. a'b' has 2 components, one along the pitman known as the normal component , $\mathbf{a} \mathbf{' a}_{\mathbf{p}}$, and the tangential component , $\mathbf{a}_{\mathbf{p}} \mathbf{b}^{\mathbf{b}}$ '.similarly $\mathbf{0} \mathbf{o}^{\prime} \mathbf{b}$ ' has 2 components ,the normal component $\mathbf{o b}{ }_{\mathbf{t}}^{\mathbf{t}}$, and the tangential component, $\mathbf{b}_{\mathbf{t}} \mathbf{b}^{\mathbf{\prime}}$.
$\beta, \alpha, t, 1$ and $\gamma$ are as per Fig. 2.1.
The angular velocityof crank is $\omega . r$ is the radius of the crank.
The coordinate of point a' would be $\left(\omega^{2} r \cos (180+\beta), \omega^{2} r\right.$ $\sin (180+\beta))$
$\omega_{\mathrm{t}}$ is the angular velocity of the toggle plate.
$\omega_{\mathrm{p}}$ is the angular velocity of the pitman.
The coordinate of point b' would be $\left(\omega^{2}{ }_{t} t \cos (\gamma), \omega^{2}{ }_{t} t \sin \right.$ $(\gamma)$ )
The coordinate of point $\mathrm{a}^{\prime}{ }_{\mathrm{p}}$ would be $\left(\left(\omega^{2} \mathrm{r} \cos (180+\beta)+\omega^{2}{ }_{\mathrm{p}}\right.\right.$ $\left.l \cos (90-\alpha)),\left(\omega^{2} r \sin (180+\beta)+\omega_{p}^{2} l \sin (90-\alpha)\right)\right)$
Noting that the line a'a' ${ }_{p}$ makes angle $\alpha$ (clockwise) with
The X axis and the line $\mathrm{b}_{\mathrm{t}}{ }^{\mathrm{b}}$ ' makes an angle $(90+\gamma)=\theta$
With the x axis their point of intersection is given by
$A_{b^{\prime} x}=\left[(w \tan \alpha+z)+\left(\omega_{t}^{2}{ }_{t} \cos (\gamma) \tan (\theta)-\omega^{2}{ }_{t} t \sin (\gamma)\right] /(\right.$ $\tan \alpha+\tan (\theta))$.
$\mathrm{A}_{\mathrm{b}^{\prime} \mathrm{y}}=\left[\{(\mathrm{w} \tan \alpha+\mathrm{z}) \tan (90+\gamma)\}-\left(\omega^{2}{ }_{\mathrm{t}} \mathrm{t} \cos (\gamma) \tan (\theta)-. \omega^{2}{ }_{\mathrm{t}} \mathrm{t}\right.\right.$ $\sin (\gamma) \tan \alpha)] /(\tan \alpha+\tan (\theta))$
Where $\mathrm{A}_{\mathrm{b} \text { 'x }}$ and $\mathrm{A}_{\mathrm{b} \text { 'y }}$ give the x and y component of acceleration of point $b^{\prime}$ and $W$ and $z$ are the $x$ and $y$ coordinates of point $a^{\prime}{ }_{p}$.

## 4. FORCE POLYGON FOR THE PITMAN



Fig. 4.1: Force polygon for the pitman.
$\mathbf{M g}$ is the force due to weight of pitman.
$\mathbf{M a}$ com is the inertia force of the pitman; $\mathbf{F}_{\text {ext }}$ is the external force acting due to crushing action. ( $\left.\mathbf{F}^{\mathbf{s}}\right)_{\mathbf{n}}$ normal reaction acting at eccentric shaft or the normal reaction at at bearing, $\left(\mathbf{F}^{s}\right)_{\mathbf{t}}$ is the force causing moment at the crank ; $\left(\mathbf{F}^{\mathbf{t}}\right)_{\mathbf{n}}$ is normal reaction at toggle plate.

Force polygon can be used for determining the two unknown forces i.e. $\left(\mathbf{F}^{\mathbf{s}}\right)_{\mathbf{n}}$ and $\left(\mathbf{F}^{\mathbf{t}}\right)_{\mathbf{n}}$.
This unknown can be easily figured out by considering the summation of all the force in x and y direction.

For ( $0 \leq \beta \leq 360$ )
In x direction,
$\mathrm{Ma}_{\text {com }} \cos \theta+\mathrm{F}_{\text {ext }} \cos \alpha_{1}+\left(\mathrm{F}^{\mathrm{s}}\right)_{\mathrm{t}} \cos (90+\beta)+\left(\mathrm{F}^{\mathrm{s}}\right)_{\mathrm{n}} \cos (\beta)-$ $\left(\mathrm{F}^{\mathrm{t}}\right)_{\mathrm{n}} \cos \gamma=0$ (4.1)

In y direction,
$-M g+M a_{\text {com }} \sin \theta-f_{\text {ext }} \sin \alpha_{1}+\left(F^{s}\right)_{\mathrm{t}} \sin (90+\beta)+\left(\mathrm{F}^{\mathrm{s}}\right)_{\mathrm{n}}$ $\sin (\beta)-\left(\mathrm{F}^{\mathrm{t}}\right)_{\mathrm{n}} \sin (\gamma)=0(4.2)$

## 5. DYNAMIC ANALYSIS OF JAW CRUSHER.

External Forces acting on the jaw crusher system is shown as in Fig. 5.1.


Fig. 5.1: External forces acting on the jaw crusher system.
Where,
$\mathbf{T}$ is the torque on the crank due to $\left(\mathbf{F}^{\mathbf{s}}\right)_{\mathbf{t}}$
$\mathbf{F}_{\text {ext }}$ is the external force acting due to crushing action
$\mathbf{M g}$ is the force due to weight of pitman
$\mathbf{M a} \mathbf{c o m}$ is the inertia force of the pitman

At any particular crank angle ( $\beta$ ),for a very small instant the net work done on the sytem by all forces is zero,
$\rightarrow \mathrm{T}^{*} \omega+\mathrm{mg}^{*} \mathrm{~V}_{\mathrm{com}}+\mathrm{F}^{*} \mathrm{~V}_{\mathrm{ext}}+\mathrm{ma}_{\mathrm{com}} * \mathrm{~V}_{\mathrm{i}}=0$
Where,

- $\quad \omega$ is the crank angular velocity
- $\quad \mathrm{V}_{\text {com }}$ is the velocity of centre of mass
- $-\mathrm{V}_{\text {ext }}$ is the velocity where the force due to crushing.
- $-\mathrm{V}_{\mathrm{i}}$ is the velocity of the point where the inertia force acts .

Torque , T , can be derived from the above equation.which can be used to calculate $\left(\mathbf{F}^{\mathbf{s}}\right)_{\mathbf{t}}$.

Using equation 4.1 and 4.2 the 2 unknown forces on the pitman i.e $\left(\mathrm{F}^{\mathrm{s}}\right)_{\mathrm{n}}$ and $\left(\mathrm{F}^{\mathrm{t}}\right)_{\mathrm{n}}$ can be found out.

## 6. CONCLUSION

The paper draws out the mathematical formulation on the analytical approach to predict the velocity and the acceleration of any point rigidly fixed to the moving jaw much needed for the inertial force calculation at various section as a function of different angle of the crank. The mathematical model successfully can be used for the forces exerted on the moving jaw by the toggle plate and the force exerted by the crank. For example, the same mathematical modelling can be used in the bearing design. This model further helps us to find out the forces and the acceleration at any point rigidly fixed to the moving jaw.

## REFERENCES

[1] James G Donovan. Fracture toughness based models for the prediction of power consumption, product size and capacity of jaw crusher, Ph.D. thesis, U.S.A, Virginia Polytechnic Institute and state University, 2003
[2] Bharule Ajay Suresh, computer aided design and analysis of . swing plate of jaw crusher, NIT Rourkela, 1-11, 2009
[3] CAO Jinxi, RONG Xingfu, YANG Shichun, jaw plate kinematical analysis for single toggle jaw crusher design, IEEE International Technology and Innovation Conference, 62-66, 2006
[4] Sobhan Kumar Garnaik, Computer Aided Design of Jaw crusher,NIT Rourkela,2010
[5] Thomas Bevan, The Theory of Machines, Third Edition
[6] S S Rattan, Theory of Machines, Third Edition
[7] Cao Jinxi, Qin Zhiyu, Wang Guopeng, Rong Xingfu, Yang Shichun, Investigation on Kinetic Features of Multi-Liners in Coupler Plane of Single Toggle Jaw Crusher.
[8] http://itools.subhashbose.com/grapher/index.php

